

Theory of Virus Public Infection Through The Weiss Approach

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Abstract

A theory of virus infection in humans by using the Weiss's approach is presented. For this end it was associated physical observables such as the distance and wind velocity. The model assumes random quantities from the fact that the confluence of healthy and infected people has not any known law. In this manner we adjudicate various probabilities that might to encompass a realistic scenario. From the resulting theoretical model, various curves of probability and infections have demonstrated that still under a scheme of care outdoor, the wind velocity can be a important factor of infections in open areas. In this manner, a single infected can transmit the strain in a radius of 5 for a wind velocity of 7m/s.

1. Introduction

The rapid spread of the so-called Covid-19 [1] has triggered the imminent employment of a plethora of mathematical machineries that target to understand the properties of this accelerated spread in people [2]. From the point of view of epidemiology the why one certain family of virus acquires the capability to infect people is because its vectorial properties that usually consists in the physical areas by which the virus can survive or adapted to climatological conditions [3]. Thus, virus might be potentially lethal if these microbiological aggregations can engage physical areas for their fast transportability due to wind. Among a plethora of ways of transmission, virus can be passed from one people to another through airborne or public air such breadth or any action that would led to people to expire millimeter drops can easily stay in air for various hours [4]. It is one of several reasons why is highly recommended to avoid public zones where there is a substantial probability that one or various infected people can transmit the virus through air. In this manner one can find that public transportation

might be also a place where infections would acquire their maximum values. Other places where accumulation of people is perceived as a potential scenario of multiple transmission are banned along the period of quarantine. Such examples are seen nowadays as part of the decisions to avoid a fast jump on the number of infections per unit of time. In this manner curfew appears as an confident action to guarantee to stop the spread. While people are avoiding physical contact, virus might be going through solid and liquids that can reach to contact entries inside the human body. Clearly, the nature of spread although not deterministic, it might be engage in part to stochastic factors in part, so that one can talk about probabilities of transmission depending entirely on physical factors. Actually all these issues emerge as relevant views in pandemic epoch in the sense that a deterministic control of spread might to restore human activities once the outbreak has passed their critic values against the human groups. Clearly from the mathematical angle one can attack the successive contagious problem through formulation either based on probabilities of direct measurements that employ so one extent realistic approximations as to data of number of infections and exposure places [5]. From the view of mathematical methodologies of public infections, one can find the formalism developed by Wells and Riley that models the probability of infections in close spaces [6] with a minimal airborne, as the cases seen in public transport (bus, train and airplanes) . Thus, in this theory the exposure time appears as a crucial variable that might define the action of contagious. It actually a function depending on a negative (or decreasing) exponential.

In this paper it was addressed the issue of multiple infections through the Weiss's theory that states that the probability of matching an empty space with a random object would have to be proportional to the logarithm of the number of spaces. Therefore, the present analysis consider the following:(i) number of infected people, (2) estimated number of infections,

(3) distances, and (4) wind velocity. Clearly, this idea fits well to the well-known epidemiological problem of successive infections in public areas. Therefore one can take advantage of the Weiss's theory to develop a formalism that targets to estimate probabilities of infection based on two observables: (i) distance and (ii) wind velocity. As dictated nowadays, the presence of Covid-19 in public areas as well as the imminent presence of bacteria in health center forces us to paid a particular attention to derive a quantitative and accurate expressions in order to establish a relation of distances and wind direction by knowing the virus aerodynamics. Thus, from the question: **What is the probability to be infected in certain distances for outdoor areas given a finite number of infected ones?**

The answer of this question is given in section-II where we have developed a theory entirely based on the Weiss formalism that consists in probabilities that depends on logarithmic formulations. In this manner in third section, the epidemiological application of the theory of Weiss is done. Finally from the results, we derive the conclusion of paper.

2. Theory Formulation

2.1. The Weiss's Formulation

The original formulation of Weiss [7] establishes that if Q objects are thrown into N cells with the golden condition: **each object is limited to fall to only one cell**. If this repeated M times, then when both N and QM becomes a large number then the probability for finding an unoccupied cell is governed by

$$P(M) = \frac{N \text{Log} N}{QM}. \quad (1)$$

Clearly, this can be formulated as written below:

$$P(M) = \text{Log} \left[(N)^{\frac{N}{QM}} \right], \quad (2)$$

exhibiting a form of type $\text{Log} N^N$ that is seen as $N \text{Log} N$ a potential expression to be perceived a Shannons entropy [8]. Although the Weiss's formulation has not this purpose, one can extend the original formulation to the side of complex systems by the which the Weiss's model would fit. In fact, although one can identify from the point of view of physics that there is actually several physical observables that would be adjusted to Eq.(1), the link of this to the entropy territory might to require of a robust support. On the other hand, the formulation of Eq.(1) lacks of a realistic association to the problem of public infections in times of pandemic. In fact, for arbitrary

values of $1/(QM)$ the term $N \text{Log} N$ turns out to be larger, fact that diverges seriously with the central point of the present study. In fact, a simple inspection to the morphology of $N \text{Log} N$ yields that only for small values of N , the square $(N \text{Log} N)^2$ can only yields values between 0 and 1, that restores the central scope of Eq.(1).

Therefore, under a fully scenario of Shannon's entropy, one can demand that:

$$\frac{N}{QM} = \ell \quad (3)$$

with ℓ an positive integer. Thus, the Shannon's entropy derived from Weiss's equation can be written as:

$$\mathcal{E} = \text{Log} [N^\ell] = \ell \text{Log} N. \quad (4)$$

Subsequently, N acquire the meaning of available cells. In praxis, this availability would depend entirely on time. Thus imply to rewrite $N \rightarrow N(t)$, that demands to change Eq.(4) in the following form:

$$\mathcal{E} = \ell \text{Log} N(t). \quad (5)$$

Thus, while $P(\ell, t)$ acquires the meaning of entropy or disorder of system, $N(t)$ is now perceived as the probability that any system has N available cells at the time t . Under the assumption that $N(t)$ is an universal function of number of available cells (or physical states), then $N(t)$ might be described by a continuous function. This can be translated as follows:

$$\mathcal{E}(\ell, t) = \ell \text{Log} \sum_j^J \mathcal{C}_j \mathcal{P}_j(t). \quad (6)$$

In this manner the Weiss's equation have been related to a phenomenon of entropy by which the number of cell has now as meaning the available cells at an instantaneous time t . A more wide analysis of the entropy interpretation of Weiss's equation is beyond the scope of this paper. Instead to see Ref.[7].

2.2. Extension and Modifications of Weiss's Equation

Turning back to Eq.(1), one can define $M = 1/\omega$ as the frequency of being in any event either is accepting objects or does not. Thus Eq.(1) is rewritten as:

$$P(\omega) = \frac{\omega N \text{Log} N}{Q}. \quad (7)$$

With this, the derivative of $P(\omega)$ with respect to ω turns out to be a constant. Eq.(2) might be derived from the Logarithm operation in the sense that.

$$P(\omega) = \frac{N}{Q} \text{Log} N^\omega. \quad (8)$$

In realistic applications, it should be noted that the fact that cells and objects are related to elements of an event or occurrence, it is entirely suitable that all these components have a random character. Therefore, the introduction of stochastic is valid and pertinent with the purpose of the present analysis. In this manner, consider the scenario that $P(\omega)$ and the rate N/Q are random numbers. Thus one can rewrite Eq.(8) as

$$P(\omega) \frac{Q}{N} = R(\omega, Q, N) = \text{Log} N^\omega. \quad (9)$$

Taking advantage that $R(\omega, Q, N)$ is defined now as a pure random number

$$R(\omega, Q, N) = \lambda \quad (10)$$

then it is plausible to define it as a product in the sense that $R(\omega, Q, N) = R_A(\omega, N)R_B(Q)$ so that

$$R_A(\omega, N)R_B(Q) = \text{Log} N^\omega, \quad (11)$$

a grouping in both sides one gets that:

$$R_A(\omega, N) = \frac{\text{Log} N^\omega}{R_B(Q)} = \lambda \quad (12)$$

because $R_A(\omega, N)$ is random, then $\lambda/R_B(\omega, N)$ is also a random number then

$$\frac{\text{Log} N^\omega}{R_B(Q)} = \lambda \quad (13)$$

and a relation for N is derived:

$$N(\omega, Q) = (\text{Exp}[\lambda R_B(Q)])^{\frac{1}{\omega}}. \quad (14)$$

A generalization of Eq.(14) is applied in Q the number of objects, that can now depend on the frequency ω , thus one can arrive to

$$N(\omega, Q) = (\text{Exp}[\lambda R_B(Q(\omega))])^{\frac{1}{\omega}}, \quad (15)$$

with λ a pure random number without any compromise to acquire a physical meaning.

3. Epidemiological Implications

One can take advantage of the implications of Eq.(15) in an entire scenario of virus infections in public spaces. In this manner one can focus the previously presented theory to the concrete case of multiple infections. For this, one can establish the following associations and illustrated in Fig.1:

- Q objects \rightarrow Q infected drops,
- N cells \rightarrow N people,
- $\omega \rightarrow$ frequency by which N people experience the arrival of Q drops,
- λ a random number that encompasses the aleatory of events.

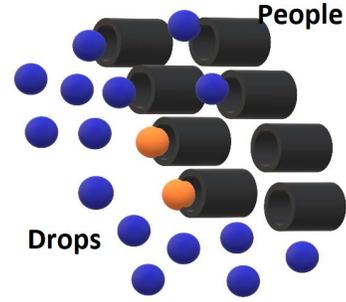


Figure 1. Sketch that relates the Weiss's equation and a toy model of infections through drops aerosol. This representation do not includes the distance between cell (people) and objects (drops). However, under this scenario distance is tacit.

Eq.(15) is interpreted defined as the probability of N people that might be under infection when are in contact with Q drops. With the restitution of time as independent variable then

$$N(T, Q) = (\text{Exp}[\lambda R_B(Q(T))])^T. \quad (16)$$

It should be noted that $R_B(Q)$ is a function entirely dependent on the number of drops. Of course in pandemic times, infection is translated in terms of drops expulsion by people that have been infected previously. Of course, the emission of drops might not be aleatory but instead it would be periodically, so that it would be adjusted to a sinusoid function in according to $R_B(Q, T) = -Q_0 \text{Sin}(Q(T))$ so that one arrives to:

$$N(T, Q) = (\text{Exp}[-\lambda Q_0 \text{Sin}(Q(T))])^T. \quad (17)$$

with Q_0 a constant. This would be depending for instance on the initial number of drops at an initial time. In this way Eq.(17) is interpreted as the number of N people that is receiving an amount Q infected drops at the time T . This is actually to some extent a bit contrary to the primary or fundamental Weiss's equation that states the probability of unoccupied cell. While Eq.(17) have been established for all those people that are receiving $Q(T)$ drops at the time T . Nevertheless, one is interested in all those people that are not receiving any drops, so that it can be established in a straightforward manner through a view associated to probabilities: $P_R + P_N = 1$ with P_R and P_N the probabilities of those are receiving and do not, respectively.

Certainly, the building of a formalism that involves the contrary case, it is the ones that might not be subject to not any infection would demand to incorporate extra

variables that put apart the risk to be under of scenario of probability of infection. It actually is perceived as the necessity to implement physical variables that break down the action of infection. As it will be seen below, the scenario by the which a physical distance among people can be modify the concept of probability of infection.

3.1. Incorporation of Distance

In indoor and outdoor environments, drop dynamics that drives the paths and physical routes of infections is dictated by wind and distances. An important assumption is that gravity is fully negligible and aerodynamics of expelled drops is dictated by wind velocity, composition and an accurate information about weight and geometry of aerosol could be crucial to establish a robust analysis in very specific models of drop propagation. Thus, a logic definition of the velocity of drop when it is emitted by a infected one in open areas, is dictated by wind velocity. Therefore it is imminently written as $V = \frac{D}{T}$ so that the distance is then $D = VT$, in turn one has that $T = D/V$. From Eq.(17) it is valid to employ the approximations given by

$$\text{Sin}Q \approx 1 + Q \quad (18)$$

$$Q(T) = -T - T^2 \quad (19)$$

with a defined restriction to $\text{Sin}Q$ in the sense that a term of type Q^2 , Q^3 and higher order might to produce a fully exponential model that would be not adjusted to a realistic applicability. With Eq.(18) and Eq.(19) then one can write down the number of drops is given by:

$$N(T, Q) = (\text{Exp}[-\lambda Q_0(1 - T - T^2)])^T. \quad (20)$$

When the criterion that T is directly proportional to the distance as written above, then one has that:

$$N(D, Q_0) = \left(\text{Exp} \left[-\lambda Q_0 \left\{ 1 - \frac{D}{V} - \left(\frac{D}{V} \right)^2 \right\} \right] \right)^{\frac{D}{V}}. \quad (21)$$

that express the number of infections given an initial of infected ones proportional to number of drops Q_0 at a distance D . It should noted that although Eq.(21) express its deterministic character, it is actually semi-random because the presence of constant λ whose origin might be correlated to physical variables. On the other side Eq.(21) can also be perceived as a function of type $N(x) = n^x(x)$ in agreement to Eq.(1) or Weiss's equation. Actually the inclusion of physical variables have turned out in a model that now depends on distance essentially. Indeed, under the

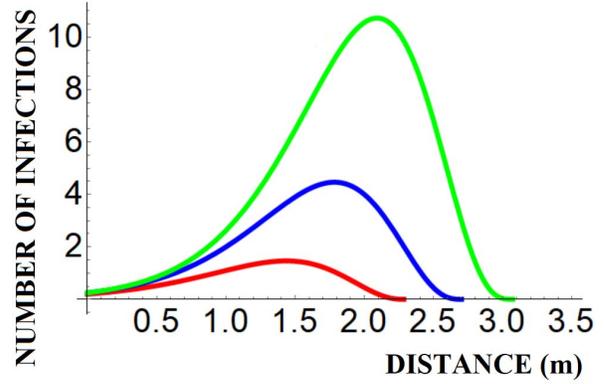


Figure 2. Number of infections caused by a single 2, 4 and 5 infected ones in colors: red, blue and green, respectively, as function of distance in meters. Curves were done with the package Wolfram [9].

approximation $e^x \approx x$ it is possible to interpret to $N(D)$ as the main function of infections as a function of distance and the initial number of infections namely $\mathbf{n} = -\lambda Q_0$,

$$N(D) = \mathbf{n} \left[\left\{ 1 - \frac{D}{V} - \left(\frac{D}{V} \right)^2 \right\} \right]^{\frac{D}{V}}. \quad (22)$$

In Fig.2 up to three different scenarios for $N(D)$ are depicted. Here, the physical interpretation to \mathbf{n} : the rate of infections over total people (assumed to be healthy). It was taken a wind velocity of order of $V = m/s$ outdoor. The color red indicates that 2 infected ones can infect a single people in a distance of 1.25m. In blue color, 3 infected one do up to 4 healthy ones at 1.5m. Critically, in green color, 5 infected ones can do at 2.25m. It was assumed that wind direction takes the path that joins spatially the position of infected and healthy ones. In Fig.3, the probability of infection for wind velocities as shown there, for $V=1m/s$, $3m/s$, $5m/s$ and $7m/s$ of a single infected one can transmit the virus to 4 people. For instance the curve corresponding to $1m/s$ exhibit that infections can be done with a high probability in a radius of $0.5m$. The curve in which wind acquires a velocity of $3m/s$ demonstrates that for this case, infections is possible in a radius of $3m$. Subsequent filled curves for wind velocities of order of $5m/s$ and $7m/s$ demonstrate that wind is a crucial factor for multiple infections as seen in the last case where infections with a probability of 50% for a distance of $6m$ can be done. It is important to note that although current schemes of prevention have been established at various countries such as the one of being 1 up to

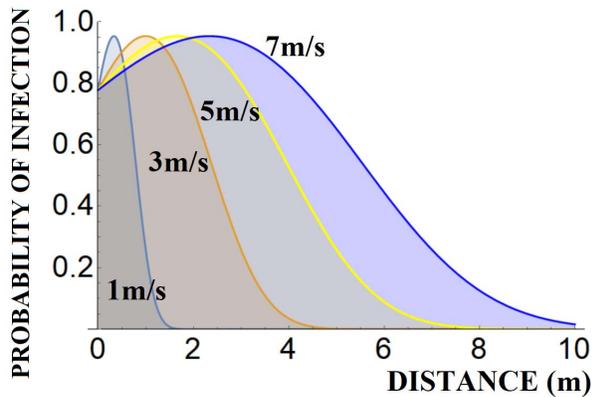


Figure 3. The probability of infections for 4 different values of wind. Theory of Weiss, exhibits the prediction that infections can be done up to 5m away from the carrier. Curves were done with the package Wolfram [9].

2 meters of distance between two individuals, it might not be safe in outdoor areas that are strongly affected by wind. Thus, for wind velocities of 7m/s a 20% of probability is predicted. Clearly, these estimations have been done with a single infected one. Once the number of infected increases, it would yield high probabilities. It should be remarked that in all paper, it was assumed that a **single drop or object** from a bunching of them expelled by a infected one is the one that impacts and **stays in mouth or eyes of the healthy ones, also called the cells.**

4. Conclusion

In this paper, it was developed a theory based on the Weiss's formulation that states the chance of any object to reach an empty cell. Thus, through derivations of logarithm expressions, it was formulated a model that has as central objective the calculation of the number of infections as well as the probability of infection as function of distance. It was assumed that in outdoor areas, drops can move on in according to wind velocities since virus can stay in air [10] and [11]. Thus, various predictions of infections have been given establishing a fine relation between the chance to be infected because wind velocities without care the distance between the infected and healthy one. Thus, special attention was paid on the distances that would reflect the more relevant parameter outdoor. Although these results are preliminary, future developments, will assume a fully physical scheme that involves the effect of temperature on a concrete virus.

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