Motivational Program Based On The Polya Method To Improve The Solving Of Mathematical Problems

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Abstract: Recent international evaluations regarding educational level have revealed that Peru is in the rankings of countries with the lowest performance in various subjects, including mathematics. The Polya method is presented as a solution to this serious situation, which assures that if its four steps are considered, better results will be obtained than the traditional method of teaching mathematics. This study narrates the application of a motivational program, in which the Polya method was applied in order to improve the solving of mathematical problem solving in the third grade of secondary school in educational institutions in Peru. The researchers identified two groups of students, one composed of 39 students in which mathematics was taught applying the traditional method (control group), and another group of students composed of 41 students in which this program was applied (experimental group). The period of this quasi-experiment covered the third quarter of school year 2019. Pre-tests and post-tests were applied to both groups. Finally, the hypothesis was contrasted by means of the chi-square test, obtaining as a result 182.142 with a confidence level of 5%, which affirms the general hypothesis formulated, that is: if the motivational program based on Polya's method is applied, then the solving of mathematical problem solving in the third grade of secondary school in Peru will be improved.

Index Terms: Motivational program, Polya method, Mathematical problems, Algebra, High school, education.

1. INTRODUCTION

EDUCATION in Peru is going through a very critical situation, reflected in the low performance it occupies in international evaluations [1], [2], it is difficult for students to discover the solution to the problem, and if they try to solve it, they do it incorrectly; since they have little capacity to identify the data, to carry out operations, and scarce capacity to formulate the answer, due in great part to the lack of analysis and understanding of the problem and to the absence of previous knowledge necessary for the solution of these. One of the great aims of education is that students learn to solve the problems of everyday life; that is why mathematics must develop in students the capacity to pose and solve problems, if we want to have productive citizens in the future. Solving a problem implies finding a path that is not known, that is, developing a strategy to find a solution, in this sense, solving problems in mathematics involves a commitment of students in ways of thinking, habits of perseverance, confidence in unknown situations, providing benefits in daily life, at work and in the scientific and intellectual field [3]. Watts, in his proposal about science teaching in secondary education [4], states that science teaching, the conception given by teachers towards the process of problem solving has been oriented by activities that only stimulate memory or the automatic repetition of certain algorithms taught for a group of type problems. In this way, the great didactic potential contained in a true solving process is wasted. In the world-wide scope the investigations carried out in Venezuela [5] show that the teaching of mathematics in its educational institutions of basic education, diversified middle and professional, has been oriented to the solving and improvement of the calculation, by means of the mechanical resolution of stereotyped exercises, transforming them into a system of definitions, rules and techniques learned memoristically by the student without reaching to understand its necessity, nor its applicability in the problematic of its surroundings.

The current reality of education in Peru is very similar to that experienced in Venezuela back in 1986, unfortunately students today try to solve a problematic situation mechanically and not heuristically. Today there are different methods that support the work of the teacher in understanding to solve mathematical problems, among them is George Polya's Method which is a strategy that improves the ability to solve mathematical problems in boys and girls, because it helps to test the curiosity of students by posing problems appropriate to their knowledge and help them solve problems through stimulating questions, thus being able to awaken a taste for independent thinking and provide certain resources for them to seek their own strategies and thus arrive at the solution [6], [7], [8]. It is important to develop in the students the abilities to identify, discover, analyze, evaluate, apply, compare, classify, predict, interpret, among others to achieve an adequate learning of the contents of the area of mathematics in the students of third grade of secondary, for which we formulate the following question: What are the effects of the application of a motivational program based on Polya's method to improve the solving of mathematical problem solving in the third grade of secondary school in Peru? The general objective is to test the effects of the application of a motivational program based on Polya's method to improve the solving of mathematical problem solving in the third grade of secondary school in Peru.

2 METHODOLOGY

A motivational program based on the Polya method was

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constructed, which was applied to one of the two classrooms of the third year mathematics course of an educational institution (called control group, sample of 39 students). It should be clarified that for the other classroom assigned to the teacher, where the third grade secondary school mathematics course is also taught, the traditional teaching method was used (called experimental group, sample of 41 students), as shown in TABLE 1.

TABLE 1
SAMPLE BY GROUP

Group	Number of students
Control	39
Experimental	41
Total	80

Elaboration: The authors.
Source: Numbers of enrollment, year 2019, of the educational institution where the study was applied

A nonprobability sample of intentional or non-random type will be used; that is to say, the experimental and control groups will be chosen, at section level, not taking into account the characteristics of the groups but determined by the results of the pre-test. Returning to the group of students who used the motivational program based on the Polya method (experimental group), the four steps of this method were applied, which were applied through strategies and measured through indicators, according to TABLE 2.

TABLE 2
POLYA METHOD STEPS

Polya Method Steps
Step 1: Understand the problem.
Step 2: Make a plan.
Step 3: Implement the plan.
Step 4: Look back.

Elaboration: The authors.
Source: The authors.

The content of the sessions developed for the third semester of studies of the mathematics course (third year of high school), defined by the syllable, is divided into seven sessions, which are described in TABLE 3.

TABLE 3
POLYA METHOD STEPS

Session	Knowledge	Session Name
1	Intervals.	I reinforce my learning about intervals.
2	Absolute value equations.	Let's analyze equations with absolute value.
3	Adding and subtracting polynomials.	Operating algebraic expressions.
4	Linear equations with a variable.	We solve problems about Linear equations with a variable.
5	System of linear equations.	We represent graphically a system of linear equations in the Cartesian plane.
6	Distance between two points on the real line.	I locate and determine the distance between two points on the real line.
7	Angles	The angles of our environment.

Elaboration: The authors.
Source: The authors.

2.1 **Activities**

The application of the motivational program based on Polya's method was oriented to improve the solving of mathematical problem solving in the third grade of secondary school, it was carried out in the following way:

a) The program had 07 sessions.

b) Each session lasted approximately 12 pedagogical hours.

c) The program began on August 6, 2019 and ended on October 9, 2019.

During the conduct of the research study, the following techniques and instruments were used to collect information:

a) Survey: applied to the student's object of study for it, a pre-test will be used and it will be the same in the post-test, measuring the dependent variable: problem solving. This pre and post-test was applied to the two groups of the program, both at the beginning and at the end of it.

- Pre-test: It is an instrument, a test that contains questions related to the subject that the person being evaluated knows, it also allows to evaluate the knowledge or knowledge previous to the application of a research work that pretends to validate the hypothesis. The pre-test is aimed at students in the third grade of secondary school in the educational institutions of northern Lima, with the objective of knowing the level they have in the ability to solve mathematical problems.

- Post-test: It is an examination that is carried out to the experimental group and to the control group with the purpose of knowing the obtained results. The post-test is directed to the students of third grade of secondary education of the educational institutions of northern Lima, having as objective to know the level that they have in the capacity of solution of mathematical problems. The problematic situations that were considered were themes that were applied during the program.

b) Evaluation: It is a dynamic, continuous and systematic process, focused on changes in behavior and performance, through which we verify the acquired achievements according to the proposed objectives.

The following general hypothesis is put forward.If the motivational program based on Polya's method is applied then the solving of mathematical problem solving in the third grade of secondary school in Peru will be improved. For data processing purposes, the general hypothesis has been subjected to the chi-square test. Minimum and maximum grades, range, arithmetic mean, median, fashion, standard deviation and coefficient of variation was analyzed. All of this was analyzed through the statistical software SPSS, version 23 in Spanish, and as an auxiliary means the Excel 2016 software.

3 RESULTS

Next, the descriptive results of both the experimental group and the control group will be shown, accompanied by their respective summaries, in addition, a general statistical comparative summary will be presented pre-test and post-test. It will also show the results of the lists of comparisons applied in the experimental group with their respective results, in addition to the summary of the observation card of the teacher who applied Polya's method. Finally, the general hypothesis

will be contrasted by means of the chi-square test.

3.1 Pre-test results

TABLE 4
STATISTICAL SUMMARY ACCORDING TO THE PRE-TEST OF THE CONTROL GROUP

Statistics	Pre-test (control group)
Minimum grade	06
Maximum grade	30
Range	24
Mean	15.88
Median	13
Mode	09
Standard deviation	6.8
Coefficient of variation	-0.417%
N	39

Elaboration: The authors.
Source: The authors.

According to TABLE 4, statistical summary according to the pre-test of the control group, to the sample of 39 students, the minimum grade obtained was: 06, the maximum grade obtained was: 30, being the range of the same: 24. The arithmetic mean is: 15.88, which we obtained by adding all the pre-test grades and dividing them among the sample (39 students). Its median is: 13, that is, the semisum of the central values. Its mode is: 9, that is, the most frequent value in the data set. The standard deviation is: 6.8. The coefficient of variation is: -0.417% is the relationship between the standard deviation (6.8) and the arithmetic mean (15.88).

TABLE 5
STATISTICAL SUMMARY ACCORDING TO THE PRE-TEST OF THE EXPERIMENTAL GROUP

Statistics	Pre-test (control group)
Minimum grade	00
Maximum grade	31
Range	21
Mean	10.39
Median	08
Mode	05
Standard deviation	8.764
Coefficient of variation	0.84%
N	41

Elaboration: The authors.
Source: The authors.

According to TABLE 5, statistical summary according to the pre-test of the experimental group, to the sample of 41 students, the minimum grade obtained was: 00, the maximum grade obtained was: 31, being the range of the same: 31. The arithmetic mean is: 10.39, the one that we obtained when adding all the grades of the pre-test and dividing them between the sample (41 students). Its median is: 08, that is, the semisum of the central values. Its mode is: 05, that is, the most frequent value in the data set. The standard deviation is: 8,746. The coefficient of variation is: 0.84% i.e. the relationship between the standard deviation (8.746) and the arithmetic mean (10.39).

TABLE 6
STATISTICAL SUMMARY ACCORDING TO THE PRE-TEST

Statistics	Pre-test (control group)
Minimum grade	00
Maximum grade	31
Range	31
Mean	13.34
Median	11
Mode	09
Standard deviation	8.259
Coefficient of variation	0.364%
N	80

Elaboration: The authors.
Source: The authors.

According to table 6, statistical summary according to the pre-test for the whole population (experimental group plus control group), for the sample of 80 students, the minimum grade obtained was: 00, the maximum grade obtained was: 31, being the range of the same: 31. The arithmetic mean is: 13.14, which we obtained by adding all the pre-test grades and dividing them among the sample (80 students). Its median is: 11, that is, the semisum of the central values. Its mode is: 09, that is, the most frequent value in the data set. The standard deviation is: 8.259. The coefficient of variation is: 0.364% i.e. the relationship between the standard deviation (8.249) and the arithmetic mean (13.14).

3.2 Post-test results

TABLE 7
STATISTICAL SUMMARY ACCORDING TO THE POST-TEST OF THE CONTROL GROUP

Statistics	Pre-test (control group)
Minimum grade	06
Maximum grade	37
Range	31
Mean	18.00
Median	15
Mode	12
Standard deviation	9.277
Coefficient of variation	0.789%
N	39

Elaboration: The authors.
Source: The authors.

According to TABLE 7, statistical summary according to the post-test of the control group, to the sample of 39 students, the minimum grade obtained was: 06, the maximum grade obtained was: 37, being the range of the same: 31. The arithmetic mean is: 18, which we obtained by adding all the grades of the post-test and dividing them among the sample (39 students). Its median is: 15, that is, the semisum of the central values. Its mode is: 12, that is, the most frequent value in the data set. The standard deviation is: 9.277. The coefficient of variation is: 0.789% i.e. the relationship between the standard deviation (9.277) and the arithmetic mean (18).

TABLE 8
STATISTICAL SUMMARY ACCORDING TO THE POST-TEST OF THE EXPERIMENTAL GROUP

Statistics	Pre-test (control group)
Minimum grade	04
Maximum grade	47
Range	43
Mean	26.24
Median	20
Mode	18
Standard deviation	13.024
Coefficient of variation	0.310%
N	41

Elaboration: The authors.
Source: The authors.

According to TABLE 8, statistical summary according to the post-test of the experimental group, to the sample of 41 students, the minimum grade obtained was: 04, the maximum grade obtained was: 47, being the range of the same: 43. The arithmetic mean is: 26.24, which we obtained by adding all the pre-test grades and dividing them among the sample (41 students). Its median is: 20, that is, the semisum of the central values. Its mode is: 18, that is, the most frequent value in the data set. The standard deviation is: 13.024. The coefficient of variation is: 0.31% i.e. the relationship between the standard deviation (13.024) and the arithmetic mean (26.24). It can be noted that the results of the evaluation of the experimental group present extremely favorable results in the post test, in comparison to the pre-test, a test that reinforces the general hypothesis: the motivational program based on the Polya Method does improve the solving of mathematical problem solving in the third grade of secondary school, as is stated in the general hypothesis.

TABLE 9
STATISTICAL SUMMARY ACCORDING TO THE POST-TEST OF THE CONTROL GROUP

Statistics	Pre-test (control group)
Minimum grade	06
Maximum grade	37
Range	31
Mean	18.00
Median	15
Mode	12
Standard deviation	9.277
Coefficient of variation	0.789%
N	39

Elaboration: The authors.
Source: The authors.

According to TABLE 9, statistical summary according to the post-test for the whole population (experimental group plus control group), for the sample of 80 students, the minimum grade obtained was: 04, the maximum grade obtained was: 47, being the range of the same: 43. The arithmetic mean is: 22.12, which we obtained by adding all the grades of the post-test and dividing them among the sample (80 students). Its median is: 18.5, that is, the semisum of the central values. Its mode is: 12, that is, the most frequent value in the data set. The standard deviation is: 11,936. The coefficient of variation is: 0.673% i.e. the relationship between the standard deviation (11.936) and the arithmetic mean (22.12).

3.3 Pre-test and post-test general statistical comparative summary

A summary of the results obtained in the pre-test and in the post-test for the total population of 80 students will be presented below (TABLE 10), in order to have an even greater visibility of the results obtained.

TABLE 10
PRE-TEST AND POST-TEST GENERAL STATISTICAL COMPARATIVE SUMMARY

Statistics	Pre-test	Post-test
Minimum grade	00	04
Maximum grade	31	47
Range	31	43
Mean	13.14	22.12
Median	11	18.5
Mode	09	12
Standard deviation	8.259	11.963
Coefficient of variation	0.364%	0.679
N	80	

Elaboration: The authors.
Source: The authors.

3.4 Chi-square test

Lines below (TABLE 11) is shown the statistical analysis of the chi-square test in order to corroborate the hypothesis: if the motivational program based on Polya's method is applied then the solving of mathematical problem solving in the third grade of secondary school in Peru will be improved.

TABLE 11
CHI-SQUARE TEST

	Value	df	Asymp. Sig. (2-sided)
Pearson chi-square	182.142[a]	30	0.002
Likelihood ratio	123.337	30	0.002
Linear-by-linear association	32.989	1	0.002
N of valid cases	80		

Elaboration: The authors.
Source: The authors.

After having processed the obtained data and applied the chi-square test, we can affirm that there is an association between the motivational program and the solving of problem solving, then we affirm also that calculated chi (182.142), is greater than chi table generally accepted for this type of statistical analysis with confidence level of 5% (3.84); we can affirm that the posed general hypothesis is tested, that is: if the motivational program based on Polya's method is applied then the solving of mathematical problem solving in the third grade of secondary school in Peru will be improved.

4 CONCLUSION

Polya's method is an innovative method of teaching mathematics, which consists of four steps for its correct application: understanding the problem, configuring a plan, executing the plan, and looking back. A quasi-experiment based on the application of a motivational program based on the Polya method to two groups of students of the third grade

secondary mathematics course in an educational institution in Peru was carried out, obtaining important results: if the motivational program based on the Polya method is applied then the solving of mathematical problem solving in the third grade of secondary school in Peru will be improved. There is no doubt that if this motivational program based on the Polya method were to be applied in all the educational centers where third year secondary mathematics is taught, it would be possible to obtain high results from the participating students, as well as those obtained in this quasi-experiment. The scientific community dedicated to teaching is invited to apply the Polya method for the teaching of mathematics, especially in the calculation section given the great advantages demonstrated in this manuscript. The authors, for their part, are currently preparing studies related to the integration of the Polya method and information and communication technologies (ICT), in order to know its validity of application in the Latin American context, and to be able to share the findings with those interested in future open access manuscripts.

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